Step-by-Step Intangibles!

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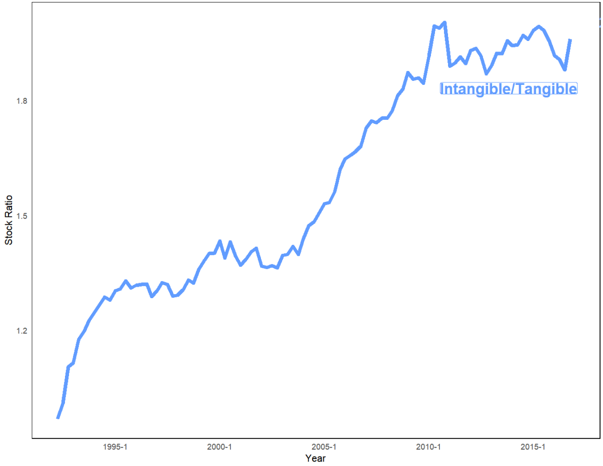
What are Intangibles and Why are They Important

- Compared to tangible assets (eg. machinery and equipment), intangible assets possess a complex (eg. scalability, sunk, spillovers, and synergies) and often invisible nature.
- These assets encompass a wide array of items, including patents, intellectual property, brand value, and organizational capital.
- My projects focus on understanding how intangible assets impact firm dynamics and growth rates, taking into account the heterogeneity of these assets.

Empirical Trend of Intangible over Tangible Ratio



Motivation

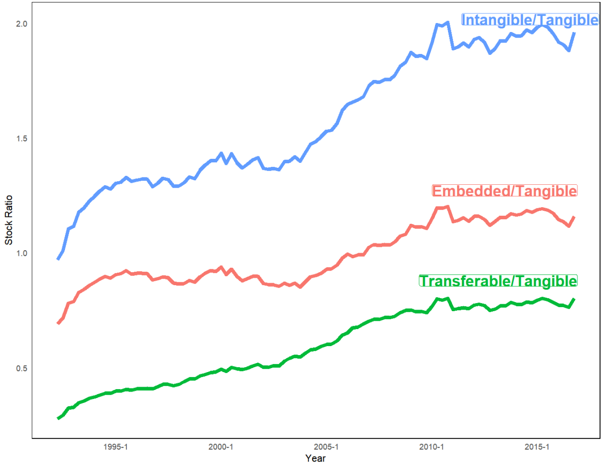
- The literature recently studied the increase of intangible assets and their impact on firm dynamics.
- However, empirical observations indicate that while the intangible-to-tangible ratio has increased since 1990, it has stagnated over the last decade.
- The motivation of this research is to explain this empirical observation and its impact on firm dynamics and growth.
- **Contribution:** The model explains the rise and plateau of the intangibles-to-tangibles ratio through the heterogeneous (Transferable/Embedded) effects of intangibles.

Transferable vs Embedded Intangibles

- I distinguish intangibles into two types: transferable (R&D, productivity) and embedded (non-transferable).
- Embedded intangible assets, such as brand value and organizational capital, are sticky on a firm and cannot be separated from it.
- On the other hand, transferable intangibles are the same as first-generation endogenous growth literature (Romer, 1990; Aghion and Howitt, 1992).

more detail

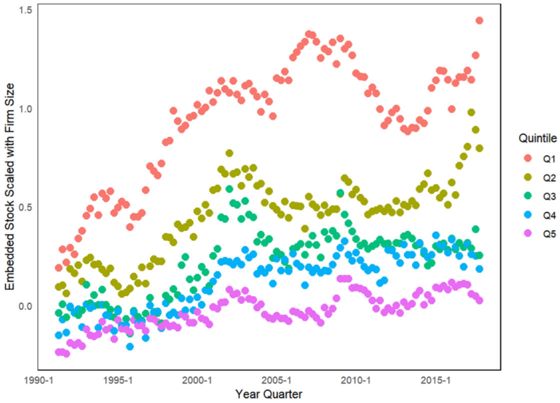
Empirical Trend of Intangibles/Tangible Ratio (II)



Intensity of Different Types of Intangibles

Log R&D (Transferable) Intensity for Each Quintile

Log Embedded Intensity for Each Quintile



Calculations

Empirical Observations

1. Smaller firms' intensity on Transferable and Embedded Stock is higher than large firms.
2. Large firms' intensity is small and stable.
3. Intangible over tangible ratio increased then stagnated.
4. Intangibles affect markup and productivity. (Crouzet and Eberly, 2019)

Preview of Predictions

1. Smaller firms have higher investment intensity on transferable and embedded stock than larger firms, while larger firms show lower investment intensity in intangibles.

2. The effect of embedded intangible assets is limited in the long run, and all three types of assets grow at the same rate in the SS.

3. In the transitional period, an increasing gap in both transferable and embedded assets within the industry reduces the demand for tangible assets.

4. When a firm adds multiple production lines, its total markup and profit rise; however, each production line's markup and profit decrease due to the span of control problem (Lucas, 1978).

Literature Review

- **Increase Markup & Market Concentration and Fall in Labor Share**: Syverson (2019, JEP), De Loecker et al. (2020, QJE), Autor et al. (2020, QJE)
- **Slowdown Business Dynamism**: Akcigit and Ates (2021, AEJ: Macro), Akcigit and Ates (2023, JPE)
- **Intangibles Effect Business Dynamism**: Crouzet and Eberly (2019, WP),Weiss (2020, WP), De Ridder (2024, AER)
- **Advertisement (Brand Value) Effect Product Perceived Quality and Increase Target Awareness**: Cavenaile and Roldan (2021, AEJ:Macro), Cavenaile et al. (2024, R&R JPE)

Summary

The literature generally shows that increasing markup, market concentration, and a rise in intangibles have a negative impact on output growth in the long run.

Model Introduction

Three types of assets: Tangible, Embedded, and Transferable.

There are two types of **sectors/lines**: final and intermediate product sectors.

Two firms compete in each intermediate goods sector, and the intermediate sector can be **unleveled** (Leader-Follower) or **leveled** (Neck-to-Neck).

A firm can produce more than one sector/line; however, due to **the span of control**, the marginal cost of producing a product in each sector/line increases.

$\Rightarrow$ The firm invests in intangible assets to achieve a competitive advantage and decrease marginal cost (price effect) over rivals, e.g., organizational capital.

Preferences and Budget Constraint

- In this economy, there is a continuous infinite horizon time with representative agents. Preferences are logarithmic, and labor supply inelastically and equal to 1.

$$\int_0^\infty e^{-\rho t} \log(C_t)\, dt$$

- Budget Constraint:

$$As\dot{s}et_t = r_t Asset_t + w_t - C_t$$

- Resource Constraint:

$$C_t + I_t^T + I_t^{Emb} + I_t^X \leq Y_t$$

- Y_t is the total output. Consumption (C_t), productive (I_t^T), embedded (I_t^{Emb}) and tangible I_t^X investment cannot exceed total output at time t.

Final Good Production Function

$$Y_t = \exp\left(\int_0^1 \log(A(\xi E_{fjt})y_{fjt} + A(\xi E_{-fjt})y_{-fjt})\, dj\right)$$

- y_{fjt} is intermediate sector/line output, $A(.)$ concave demand shifter, $\xi \in (0,1)$
- $\xi E_{fjt} = \xi \frac{e_{fjt}}{e_{fjt}+e_{-fjt}}$ shows firm i relative brand value and e_{fjt} shows firm f embedded intangible asset level.
- The final good sector gets the benefit of increasing the relative brand value because their perceived benefit (quality) from firm f is higher than $-f$.
- Each production line j is produced by a single firm f, and a single firm may own multiple active production lines $n_f = |J_f| \in Z_+$.

Final Good Sector

- **Firm Maximization Problem**

$$max_{y_{fjt}} \exp\left(\int_0^1 \log(A(\xi E_{fjt})y_{fjt} + A(\xi E_{-fjt})y_{-fjt}) \, dj\right) - \int_0^1 (p_{fjt}y_{fjt} + p_{-fjt}y_{-fjt})dj$$

- From FOCs, $y_{fjt} = \frac{Y_t}{p_{fjt}}$

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Assumption

Firms cannot internalize the demand effect. They invest in embedded intangible assets, and some of these investments have a positive spillover effect on final goods producers by increasing perceived quality.

Intermediate Sector

$$min_{x_{fjt},l_{fjt}}(r_t+\delta)x_{fjt}+w_t l_{fjt}$$

$$q_{fjt}x_{fjt}^{\alpha}l_{fjt}^{1-\alpha}\psi((1-\xi)E_fjt,n_f)^{1-\alpha}\leq y$$

- $\frac{\partial\psi(.)}{\partial n_f}<0$, when the leader has more production line marginal cost advantage decreases because the span of control and tangible capital accumulation $\dot{x}_{fjt}=I_{fjt}^{x}-\delta x_{fjt}$

  Item $(1-\xi)E_{fjt}$ shows a firm's relative organizational capital. A firm's embodied employee talent/management skills are equal to its organizational capital ratio with its rival.

Intermediate Sector (II)

$$MC_{fjt} = \left(\frac{r_t + \delta}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha}} \frac{1}{q_{fjt}}$$

- In each sector, there are two firms competing with each other a la Bertrand and $f \neq -f$.

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- In each sector, there are two firms competing with each other a la Bertrand and $f \neq -f$.
- If $q_{fjt}\,\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha} > q_{-fjt}\,\psi((1-\xi)E_{-fjt}, n_{-f})^{1-\alpha}$ I will call firm f leader and $-f$ follower.
- $q_{fjt}\,\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha} = q_{-fjt}\,\psi((1-\xi)E_{-f}, n_{-f})^{1-\alpha}$ there is a neck to neck competition

Intermediate Sector (III)

- Under a la Bertrand competition, only the leader supplies goods in each production line and $p_{fjt} = MC_{-fjt}$

$$\pi_{fjt} = \left[1 - \frac{\psi((1-\xi)(1+\theta^{-k}), n_{-f})^{1-\alpha}}{\psi((1-\xi)(1+\theta^{k}), n_{f})^{1-\alpha}} \frac{1}{\lambda^m}\right] Y_t$$

$$\mu_{fjt} = \frac{p_{fjt}}{MC_{fjt}} = \frac{\psi((1-\xi)(1+\theta^{k}), n_{f})^{1-\alpha}}{\psi((1-\xi)(1+\theta^{-k}), n_{-f})^{1-\alpha}} \lambda^m$$

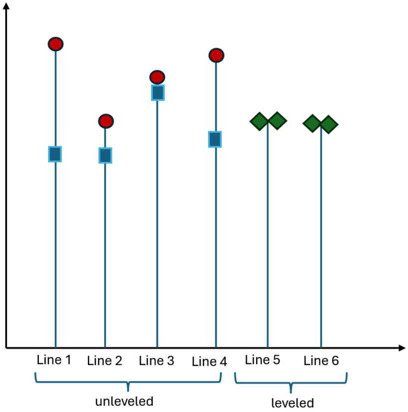
- Profit and markup reduced transferable and embedded intangible gap and # of production lines gap (**Prediction 4**)

$$\pi_f = \sum_{j \in J_f} \pi_j, \quad \mu_f = \sum_{j \in J_f} \frac{p_j}{MC_j}$$

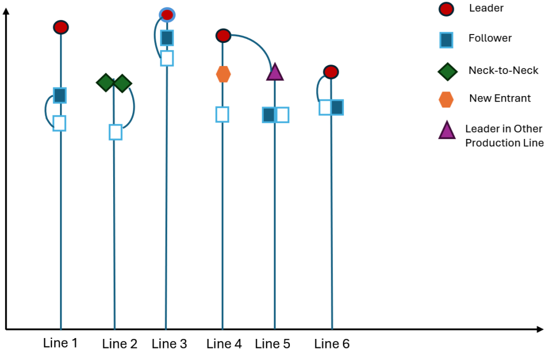
details on gaps

Multiple Scenario in Competition

$$\tau_{ijt} \equiv q_{ijt}\psi((1-\xi)' E_i, n)^{1-\alpha}$$



Production Lines



Entry, Exit and Leadership

Transferable and Embedded Investment

$$z^{Emb}_{f,j,t} = \phi(I^{Emb}_{f,j,t}) \quad \Rightarrow \quad I^{Emb}_{f,j,t} = G(z^{Emb}_{f,j,t})$$

$$z^{Int}_{f,j,t} = \phi(I^{Int}_{f,j,t}) \quad \Rightarrow \quad I^{Int}_{f,j,t} = G(z^{Int}_{f,j,t})$$

$\phi(.)$ is continuously twice differentiable, satisfy $\phi^{'}(.) > 0, \phi^{'}(.) < 0$ and $\phi(0) < \infty$. Inverse function $G(.)$ satisfy twice differentiable and $G^{'}(.) > 0, G^{''}(.) > 0$.

$$I^{Ex}_{f,j,t} = \tilde{G}(z^{Ex}_{fjt}, n)\mathbb{1}\left\{ \sum_{j=1|j\in J_f}^{n+1} \pi_j(n+1) \geq \sum_{j=1|j\in J_f}^{n} \pi_j(n) \right\}$$

$\frac{\partial \tilde{G}(.)}{\partial n} > 0$ (**Prediction 1 and Emp Obsv 2 and 3**), $I^T_{f,j,t} = I^{Int}_{f,j,t} + I^{Ex}_{f,j,t}$.

Markov Perfect Equilibrium

- Transferable and embedded intangibles gap $\bar{m}, \bar{k}$ and $\#$ of production line gap n bounded far future limit. The joint distribution of transferable and embedded intangibles gap and number of production line gap n defined,

$$\sum_{m=0,k=0,n=0}^{\bar{m},\bar{k},\bar{n}} \mu_{m,k,n}(t) = 1$$

- Transferable and embedded intangible gap, and n sufficient to define payoff of Markov Perfect Equilibria and MPE natural solution to the model. Game consist of

$$\Gamma_{m,n,k,t} = \{z^{I}_{i,j,t}, z^{E}_{i,j,t}, z^{Emb}_{i,j,t}, p_{i,j,t}, y_{i,j,t}\}$$

- and Markov Perfect Equilibria represents time paths $\Gamma^*(t), w^*(t), r^*(t), Y^*(t), X^*(t)$

Dynamics

Proposition

- Under constant embedded intangible assets gap $\bar{k}$ and production line gap $\bar{n}$ with each one-step productivity gain, the difference in value functions decreases

$$v_{m+1,\bar{k},\bar{n}} - v_{m,\bar{k},\bar{n}} > v_{m+2,\bar{k},\bar{n}} - v_{m+1,\bar{k},\bar{n}} \; \forall m \geq 1$$

- As the leader increases its productivity level by one more step, its investment incentive decreases.

$$v_{\bar{m},k+1,\bar{n}} - v_{\bar{m},k,\bar{n}} > v_{\bar{m},k+2,\bar{n}} - v_{\bar{m},k+1,\bar{n}} \; \forall k \geq 1$$

- An increase in the embedded gap decreases the incentive for investment in embedded intangible assets.

Transitional Period

$$\Upsilon_t = \int_0^1 \ln\left(\frac{A\left(\xi\left(1+\theta^{\sum_m\sum_k\sum_n k\,\mu(t)}\right)\right)\psi\left(\left(1-\xi\right)\left(1+\frac{1}{\theta^{\sum_m\sum_k\sum_n k\,\mu(t)}}\right), n_{-f}\right)^{1-\alpha}}{\chi\,\lambda^{\sum_m\sum_k\sum_n m\,\mu(t)}}\, q_{ijt}\right) dj$$

Steady State

In steady state χ^* grow constant and

$$\ln\Upsilon_t = \ln\left(\frac{A\left(\xi\left(1+\theta^{\sum_m\sum_k\sum_n k\,\mu^*}\right)\right)\psi\left(\left(1-\xi\right)\left(1+\frac{1}{\theta^{\sum_m\sum_k\sum_n k\,\mu^*}}\right), n_{-f}\right)^{1-\alpha}}{\chi^*\,\lambda^{\sum_m\sum_k\sum_n m\,\mu^*}}\right) + Q_t$$

$Q_t = \int_0^1 \ln\, q_{ijt}\, dj$. Now if we take logarithm of both sides and derivative w.r.t t,

$$g^* = \frac{\dot{\Upsilon}_t}{\Upsilon_t} = \frac{\dot{Q}_t}{Q_t}$$

Conclusion

- The engine of growth is transferable intangibles and embedded intangible assets can only be affected during the transitional period.
- In the steady state, three types of assets—tangible, transferable, and embedded—grow at the same rate.
- When the firm gets new production lines, its total markup and profit rise; however, each production line's markup and profit decrease due to the span of control.

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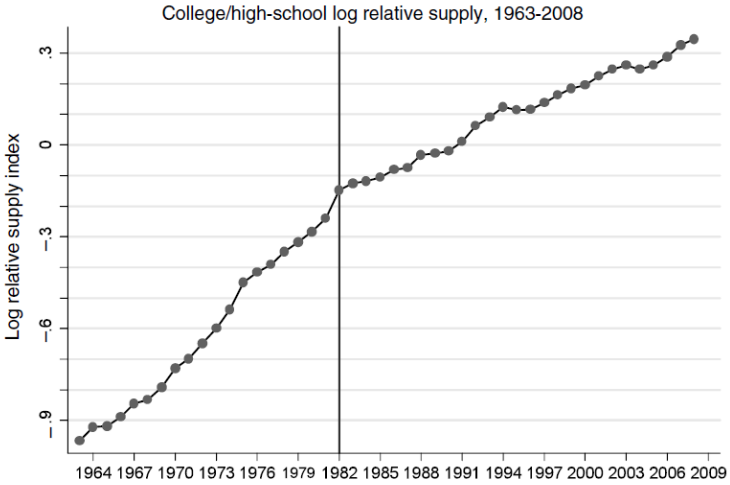
Future Agenda:

1. Perform model simulations and calibrations
2. Validate markup predictions with empirical evidence using Compustat Segment Data.

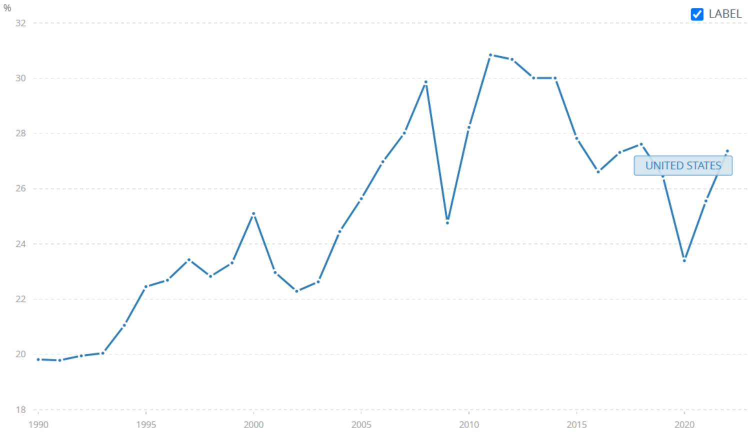
Extension

- What caused the increase in intangible assets since 1990?
- According to Melitz and Redding (2023), Trade affects innovation:
  1. **Market Size**, 2. **Competition**, 3. Spillover, 4. Comparative Advantage
- One possible explanation is that globalization and skill-biased technological changes increase the demand for product differentiation and task specialization.
- High competition within the same production line makes marginal cost advantages crucial.
- Moreover, task specialization becomes crucial for firms' investment in organizational capital due to increased market size and skill-biased technological development.



Source: Acemoglu and Autor (2011)

Trade % GDP



Source: World Bank

Model Introduction

- There are two types of labor, high-skill and low-skill, both supplied inelastically.
- Firms can operate only in one production line and the leader can be only one step ahead of the follower.
- Two types of goods: Goods produced with high-skill and low-skill labor.
- The key assumption is that goods produced with low-skill labor do not require embedded intangibles.

Final Good Sector

$$Y_t = \left(Y_{H,t}^{\sigma} + Y_{L,t}^{\sigma}\right)^{\frac{1}{\sigma}}$$

High and Low Type Aggregator

$$Y_{H,t} = \exp\left(\int_0^1 \ln\left(A\left(\xi E_i^h\right) y_{ijt}^h\right)\right)$$

$$Y_{L,t} = \exp\left(\int_0^1 \ln\left(y_{ijt}^l\right)\right)$$

Intermediate Good Sector

$$\min_{x_{ijt}, h_{ijt}} (r_t + \delta) x_{ijt} + w_t^h h_{ijt} \quad s.t$$

$$q_{ijt} x_{ijt}^{\alpha} h_{ijt}^{1-\alpha} \psi\left(\left(1-\xi\right) E_i^h\right)^{1-\alpha} \leq y^h$$

$$\min_{x_{ijt}, l_{ijt}} (r_t + \delta) x_{ijt} + w_t^l l_{ijt} \quad s.t$$

$$q_{ijt} x_{ijt}^{\alpha} l_{ijt}^{1-\alpha} \leq y^l$$

Relative Prices of High and Low Types

$$\frac{p_H}{p_L} = \left(\frac{x^h_{ijt}}{x^l_{ijt}}\right)^{2\alpha} \left(\frac{h_{ijt}}{l_{ijt}}\right)^{2(1-\alpha)} \frac{A(\xi E^h_i)}{\psi((1-\xi)E^h_i)^{(1-\alpha)}}$$

- If $2(1-\alpha) \geq 1$, increasing the relative supply of high-skill labor will make goods produced with high-skill labor more profitable.
- I introduce competition based on Aghion et al. (2005). If the sector is unleveled, the leader has no incentive to collaborate with rivals on prices, $\pi^f_1 > 0, \pi^f_{-1} = 0$ where $f =${High, Low }.
- If the sector level, then firms' incentive to collude, $\pi^f_0 = (1-\Delta)\pi^h_1, \frac{1}{2} \leq \Delta \leq 1$. Here, Δ shows product market competition and $(1-\Delta)$ fraction of leader's profit that the leveled firm can attain through collusion.

Conclusion and Summary of the Model

- To understand the effect of skill-biased technological change on intangible assets, shift $\frac{h}{l}$
- To understand the globalization effect shift Δ to increase competition, and shift Y_t to increase market size.

Value Functions

APPENDIX

Transferable Stock:

$$K_{i,t+1} = (1-\delta^K)K_{i,t} \times \frac{P^K_{t+1}}{P^K_t} + 0.5 \times I^K_{i,t}$$

$$K_{i,0} = \frac{I^K_{i,0}}{g^K + \delta^K - \pi^K(1-\delta^K)}$$

- δ^K shows depreciation rate and equal to 0.15. P^K_{t+1} price deflator for R&D (Nonresidential Invetment on Intellectual Property for R&D deflator from FRED).
- $I^K_{i,t}$ is R&D investment (xrdq item in compustat).
- π_K shows average price growth rate and g^K is average R&D growth rate in two-digit industries.
- $I^K_{i,0}$ shows when the R&D expenses of firm i first appear in the Compustat.

Back

Embedded Stock:

$$E_{i,t+1} = (1-\delta^E)E_{i,t} \times \frac{P^E_{t+1}}{P^E_t} + 0.3 \times I^E_{i,t}$$

$$E_{i,0} = \frac{I^E_{i,0}}{g^E + \delta^E - \pi^E(1-\delta^E)}$$

- δ^K shows depreciation rate and equal to 0.20. P^E_{t+1} price deflator for Embedded intangibles (Nonresidential Investment on Intellectual Property deflator from FRED).
- $I^E_{i,t}$ is embedded investment (xsgaq-xrdq item in compustat).
- π^E shows the average price growth rate, and g^E is the average (xsgaq-xrdq) growth rate in two-digit industries.
- $I^E_{i,0}$ shows when the (xsgaq-xrdq) expenses of firm i first appear in the Compustat.

Back

Transferable vs Embedded Intangibles (II)

- **Transferable Intangibles**: Patent, intellectual property, software, etc.

- **Embedded Intangibles** = Brand Value + Organizational Capital

  ⇒ Organizational Capital: Embodied employee key talent / Management Capacity / Specialized tasks

  ⇒ Firm's management skills, workforce training, work design, and embodied employee key talents and their future profitability in the production process (Carlin et al., 2012; Eisfeldt and Papanikolau, 2013; Prescott and Visscher, 1980; Van Reenen, 2004).

  ⇒ Brand Value: Increase product perceived quality (Cavenaile and Roldan, 2021)

back

Transferable and Embedded Intangibles Improvements

- The specified level technology evolves with $q_{fjt} = \lambda^{m_{fjt}} \, q_{fj0}$ and $q_{fj0} = 1$ is initial productivity level and m_{fjt} shows number of innovations and $\lambda \geq 1$.

$$q_{fj(t+\Delta t))} = \lambda q_{fjt}$$

$$\frac{q_{fjt}}{q_{-fjt}} = \frac{\lambda^{m_{fjt}}}{\lambda^{m_{-fjt}}} = \lambda^{m_{fjt} - m_{-fjt}} = \lambda^{m}$$

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$$\frac{q_{fjt}}{q_{-fjt}} = \frac{\lambda^{m_{fjt}}}{\lambda^{m_{-fjt}}} = \lambda^{m_{fjt}-m_{-fjt}} = \lambda^{m}$$

- With the same style of productivity e_{fjt} shows embedded intangible level of the firm and evolve $e_{fjt} = \theta^{k_{fjt}} e_{fj0}$ and $e_{fj0} = 1$ initial embedded value and $\theta > 1$. Embedded value gap expressed with,

$$\frac{e_{fjt}}{e_{-fjt}} = \frac{\theta^{k_{fjt}}}{\theta^{k_{-fjt}}} = \theta^{k_{fjt}-k_{-fjt}} = \theta^{k}$$

back

Transitional Period

$$\ln \Upsilon_t = \int_0^1 \ln\big(A(\xi E_i) y_{jt}\big)\, dj$$

$$y_{ijt} = q_{ijt} x_{ijt}^{\alpha} l_{ijt}^{(1-\alpha)} \psi\left((1-\xi)E_i, n_f\right)^{1-\alpha} = \frac{\Upsilon_t}{p_{ijt}} = \frac{\Upsilon_t}{\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{\psi\left((1-\xi)E_{-i}, n_{-f}\right)^{1-\alpha}} \frac{1}{q_{-ijt}}}$$

$$x_{ijt}^{\alpha} l_{ijt}^{(1-\alpha)} \psi\left((1-\xi)E_i, n_f\right)^{1-\alpha} = \frac{\Upsilon_t\ \psi\big((1-\xi)E_{-i}, -n\big)^{1-\alpha} \lambda^{-m}}{\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}}$$

Let's define $\chi = \frac{\Upsilon_t}{\left(\frac{r_t}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}}$ and write output,

$$\ln \Upsilon_t = \int_0^1 \Big(\ln A(\xi E_i) + \ln\big(q_{ijt}\big) + \ln \lambda^{-m} + (1-\alpha)\ln \psi\big((1-\xi)E_{-i}, -n\big) - \ln \chi\Big) dj$$

$$= \int_0^1 \Big(\ln A\big(\xi(1+\theta^{k})\big) + \ln q_{ijt} + \ln \lambda^{-m} + (1-\alpha)\ln \psi\big((1-\xi)(1+\theta^{-k}), -n\big) - \ln \chi\Big) dj$$

$$= \int_0^1 \ln\left(\frac{A\left(\xi\left(1+\theta^{\sum_m \sum_k \sum_n k\, \mu(t)}\right)\right) \psi\left((1-\xi)\left(1+\frac{1}{\theta^{\sum_m \sum_k \sum_n k\, \mu(t)}}\right), -n\right)^{1-\alpha}}{\chi\, \lambda^{\sum_m \sum_k \sum_n m\, \mu(t)}}\, q_{ijt}\right) dj$$

back

Creative Destruction of Leaders in Other Industry

- **Leaders take new production line**: when a firm f successfully make innovation with flow rate $z^{Ex}_{f,j^{'},t}$ randomly in any production line $j^{'}$, enter the industry and become a new producer if

$$p_f^{Ex} = \mathbb{P}\left\{ E_{fjt} \geq (E_{fj^{'}t}) \right\}$$

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$$p^{Ex}_f = \mathbb{P}\left\{ E_{fjt} \geq (E_{fj^{'}t}) \right\}$$

- Firm f embedded intangible level gap in sector $j^{'}$ be E_{fjk}.

- If firm f average embedded level gap in production line j is higher, the probability of getting a new production line increases.

back

Leader Value Function:

$$\rho v^f_{1,1} - \dot{v}^f_{1,1} = \max_{z^I_{1,1}, z^E_{1,1}} \left\{ \pi^f_1 - G\left(z^I_{1,1}\right) - G\left(z^{Emb}_{1,1}\right) + \left(z^I_{1,1} + z^E_{1,1}\right)\left[v^f_{1,1} - v^f_{1,1}\right] + \left(z^I_{-1,-1} + z^{Emb}_{-1,-1}\right)\left[v^f_{0,0} - v^f_{-1,-1}\right] \right\}$$

Follower Value Function:

$$\rho v^f_{-1,-1} - \dot{v}^f_{-1,-1} = \max_{z^I_{-1,-1}, z^E_{-1,-1}} \left\{ \pi^f_{-1} - G\left(z^I_{-1,-1}\right) - G\left(z^{Emb}_{-1,-1}\right) + \left(z^I_{-1,-1} + z^{Emb}_{-1,-1}\right)\left[v^f_{0,0} - v^f_{-1,-1}\right] + \left(z^I_{1,1} + z^{Emb}_{1,1}\right)\left[v^f_{1,1} - v^f_{1,1}\right] \right\}$$

Neck-to-Neck Competition Value Function:

$$\rho v^f_{0,0} - \dot{v}^f_{0,0} = \max_{z^I_{-1,-1}, z^E_{-1,-1}} \left\{ \pi^f_0 - G\left(z^I_{0,0}\right) - G\left(z^{Emb}_{0,0}\right) + \left(z^I_{0,0} + z^E_{0,0}\right)\left[v^f_{1,1} - v^f_{0,0}\right] + \left(z^I_{-0,-0} + z^{Emb}_{-0,-0}\right)\left[v^f_{-1,-1} - v^f_{-0,-0}\right] \right\}$$

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