Step-by-Step Intangibles!

Preliminary Draft (Short Version)

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Abstract

This paper proposes a unified explanation for the initial increase and subsequent stagnation of the intangible-to-tangible asset ratio, the decline in business dynamism, and the rise in markups. I distinguish between two types of intangible assets: transferable (R&D) and embedded (brand value and organizational capital), each influencing business dynamics differently. The model shows that an increase in embedded intangible assets during the transitional period mainly suppresses the demand for tangible assets, thereby increasing the intangible-to-tangible ratio. In the long run, embedded intangible assets have a level effect rather than a growth effect on output. Conversely, transferable intangible assets influence both the transitional and long-run periods. Additionally, markups increase with both embedded and transferable intangible assets; however, each production line's markup and profit decrease due to the hypothesis that firms suffer from a span of control problem.

Keyword: Schumpeterian growth, step-by-step innovation, intangible assets, firm dynamics

JEL Classification: E22, O31, O32, O33,034

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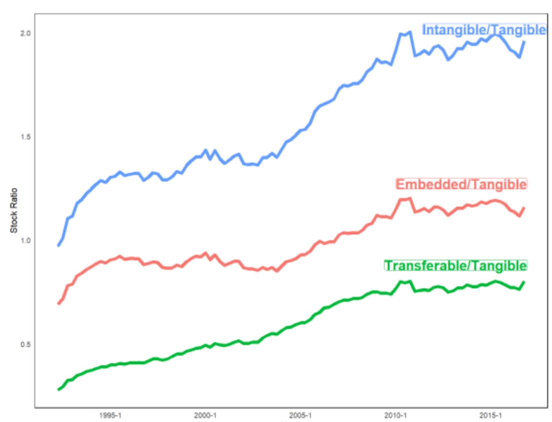
Introduction

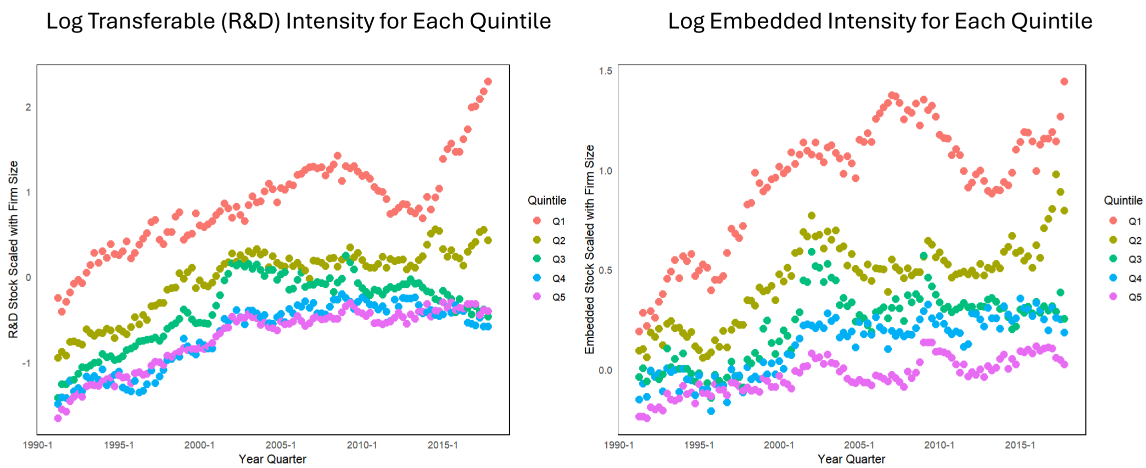
The rise in markup (De Loecker et al., 2020), decline in business dynamism (Akcigit and Ates, 2021), and slowdown in productivity growth (Fernald and Jones, 2014) have coincided with an increasing ratio of intangible to tangible assets (Figure 1). This paper proposes a unified explanation linking changes in markup, business dynamism, productivity growth, and intangible over tangible ratio with the increase of intangible assets. First-generation endogenous growth models emphasize the non-rival and limited excludability properties of intangibles (Romer, 1990; Aghion and Howitt, 1992) and these models primarily focus on types of intangible assets that affect productivity and economic growth. However, the heterogeneity of intangible assets is essential to understanding their interaction with tangible assets and, consequently, their effect on business dynamism. This heterogeneity arises from their transferability. This heterogeneity arises from the transferability of assets: if assets are transferable, I refer to them as "transferable"; otherwise, I refer to them as "embedded". Transferable intangible assets align with the intuition of the first generation of endogenous growth models. Intangibles like patents and software are transferable from firm to firm and serve as engines of growth. Conversely, non-transferable (embedded) intangible assets, such as brand value and organizational capital, are firm-specific and primarily aim to achieve a comparative advantage rather than increase productivity. To motivate this project, Figures 1 and 2 present stylized facts regarding the ratio of intangible to tangible assets and the intensity of intangible assets, including their heterogeneity.

Figure 1: Intangible over Tangible Ratio



Notes: Author's calculation. Blue line shows Intangible Assets (Embedded + Transferable Intangible Stock) divided by Tangible assets. Orange line shows only Embedded capital divided by Tangible assets and green line shows Transferable Stock divided by Tangible assets. To calculate the stock of transferable intangibles, I consider 50% of R&D expenses (xrdq in Compustat) as an investment in transferable intangibles. For embedded intangibles, I take 30% of (xsgq - xrdq) as an investment in embedded intangible assets. Tangible assets are measured as net plant, property, and equipment (ppentq). The detailed methodology for calculating intangible and tangible assets is discussed in Appendix B.

Figure 2: Transferable (R&D) and Embedded Stock Scaled with Firm Size



Notes: Author's calculation. The left figure shows the intensity of transferable intangibles, and the right figure shows the intensity of embedded intangibles. Firms are split in each quantile according to their size (revenue) value; after that, I divide each firm's transferable and embedded capital stock with its revenue to calculate intensity. This measurement gives firm intensity levels on transferable and embedded intangible assets. Finally, I take the logarithm of each year-quarter's average value. To calculate the stock of transferable intangibles, I consider 50% of R&D expenses (xrdq in Compustat) as an investment in transferable intangibles. For embedded intangibles, I take 30% of (xsgq - xrdq) as an investment in embedded intangible assets. Tangible assets are measured as net plant, property, and equipment (ppentq). The detailed methodology for calculating intangible and tangible assets is discussed in Appendix B

Empirical Observation Figure 1 illustrates that the ratio of intangible assets to tangible assets has been increasing over time, with a notable stagnation observed in the last decade. This trend is not solely driven by transferable assets; embedded intangible assets also contribute significantly to this increase. In Figure 2, I split firms into quintiles based on their size, which is represented by firm revenue. The vertical axis shows the intensity of transferable and embedded intangible assets. Intensity refers to the amount of these assets relative to firm size. I divided firms' transferable and embedded assets by their size to calculate intensity. The analysis reveals that the bottom quintile exhibits high intensity in both transferable and embedded stock, whereas this intensity remains low and stable for the top quintile. Here is a summary of empirical observation, that I leverage to motivate and discipline the model I introduce:

1. The intangible over tangible ratio has increased and stagnated in the last decade.
2. Smaller firms' intensity on transferable and embedded assets is higher than large firms.
3. Large firms' intensity on transferable and embedded assets is small and stable.

Contribution In this project, I develop a micro-founded endogenous growth model based on Akcigit and Ates (2023) to shed light on the impact of intangible assets on firm dynamics, markup, and growth rates, guided by empirical observations ($1-3$). In the model, embedded intangible assets influence the demand effect through brand value and the supply effect through organizational capital. Increasing the relative embedded intangibles sharpens the price effect compared to transferable intangible assets. Therefore, when firms increase their embedded intangible assets relative to their rivals, the price effect suppresses output and causes an increase in the intangible-to-tangible ratio during the transitional period. In the steady state, embedded intangible assets have zero impact on output growth, while transferable intangible assets are engines of growth in both the transitional and long-run periods. When firms add multiple production lines, their total markup and profit increase with both intangible assets. In this case, under the hypothesis that span of control issues in the spirit of Lucas (1968) exist, markup and profits accrued to a firm from each production line decrease.

Taxonomy of Intangibles I distinguish intangible assets between transferable[3] and embedded intangibles, categorizing each type according to their supply-and-demand effects. Transferable intangible assets like software and intellectual property (IP) affect both supply and demand side. Their distinct characteristics enable direct transfer or spillover to other companies. For instance, each patent and innovation build on prior innovations, ensuring its contribution to future advancements remains even if the originating firm ceases to exist. Embedded intangible assets, primarily brand value and organizational capital, are inherently tied to a firm and cannot be separated. While a firm needs to double its tangible capital when it doubles its output, it does not need to double its brand value or organizational capital. This characteristic renders embedded intangible assets similar to transferable intangibles. However, if a firm exits the market, the value of its embedded intangibles becomes a sunk cost for the economy. The primary reason is that embedded intangibles are typically impossible to sell or transfer in the secondary market, unlike transferable assets.

[3] I use transferable intangible assets, productivity, and R&D interchangeably during the text.

Table 1: Classification of Intangibles

	Supply	**Demand**
Transferable	Software, IP	Software, IP
Embedded	Organizational Capital	Brand Value

In this project, brand value is a demand shifter, influencing the perceived quality of a firm's products (Cavenaile and Roldan, 2021; Cavenaile et al., 2023). Moreover, the literature suggests that brand value significantly impacts target marketing by enhancing consumer awareness, and firms make extensive investments in advertising to reach consumers effectively (Cavenaile et al., 2023; Baslandze et al., 2023). Conversely, in this project, organizational capital is interpreted as encompassing management skills only affecting the supply side. This definition of organizational capital pertains to the firm's embodied employee talents and their impact on future profitability in the production process (Carlin, Chowdhry, and Garmaise, 2012; Eisfeldt and Papanikolaou, 2013; Prescott and Visscher, 1980; Van Reenen, 2004).

The paper proceeds as follows. Section 2 presents the theoretical model and discusses the mechanism. Section 3 shows an extension of the model, and Section 4 concludes.

Theoretical Model

Preferences

The theoretical model builds on Akcigit and Kerr (2018) and Akcigit and Ates (2021). In the model, the main motivation is that intermediate sector firms invest in intangible assets to enhance their competitive advantage within industries while simultaneously aiming to increase their production lines. Intangible assets are key components for both increasing competitive advantage and expanding production lines. In this economy, there is a continuous infinite horizon time with representative agent:

$$\int_0^\infty e^{-\rho t} \ln(C_t)\, dt,$$

and labor supply inelastically equal to 1. The budget constraint expressed as follows,

$$\dot{Asset}_t = r_t Asset_t + w_t - C_t.$$

Here, $Asset_t$ and C_t represent asset and consumption, respectively in time t, w_t shows wage rate of labor and r_t is interest rate. The total assets of household equal to sum of all firms' value

$$Asset_t = \int_F V_f \, df$$

where F shows set of firms in the economy. The transversality and No-Ponzi condition satisfies,

$$\lim_{t\to\infty} Asset_t \exp\left(-\int_0^t r_t \, dt\right) = 0.$$

In this particular economy, assets are the only components that can be carried over to the next period. Each consumer adopts a balanced portfolio of shares of all firms when making their asset purchases, resulting in no uncertainty within the economic system. Each period, households must decide how much to consume and save as assets for the next period. Conversely, firms must invest a portion of this capital in machines (tangible assets) to produce intermediate goods and decide how to allocate the remaining capital between embedded and transferable intangible assets. Resource constraint in this economy expressed as follows,

$$C_t + I_t^T + I_t^{Emb} + I_t^X \leq \Upsilon_t$$

consumption (C_t), expenditure on transferable intangibles (I_t^T), expenditure on embedded intangibles $\left(I_t^{Emb}\right)$, and expenditure on tangible capital ($I_{i,j,t}^X$) cannot exceed the total output Υ_t at time t.

Final Good Sector

The final good producer demands intermediate goods from a continuum of production lines [0,1] and produces final goods in a competitive market. In each production line, two firms compete with each other and $-f$ describes firm f's rivals in that production line. The final good sector production technology equal to,

$$\Upsilon_t = \exp\left(\int_0^1 \ln\left(A(\xi E_{fjt}) y_{ijt} + A(\xi E_{-fjt}) y_{-fjt}\right) dj\right).$$

In this production function y_{fjt} and y_{-fjt} represent output of the production line for firm f and $-f$ in line j at time t. The term $A(\xi E_{fjt})$ is an endogenous and concave demand shifter satisfying $A'(.) > 0, A''(.) < 0$. In the demand shifter, e_{fjt} represents firm f's embedded intangible assets in production line j and $e_{fjt} + e_{-fjt}$ represents total embedded intangible assets level of the industry. Firms play a kind of zero-sum game with embedded intangible assets, which means that when firm f increases its level of embedded intangible assets, firm $-f$'s level decreases. The term $\xi E_{fjt} = \xi \frac{e_{fjt}}{e_{fjt}+e_{-fjt}}$ indicates a fraction $\xi \in (0,1)$ of the relative embedded intangible assets is associated with relative brand value. If the relative brand value of firm f increases, the perceived benefit (quality) from firm f's product in line j for the final goods producer will be higher compared to firm $-f$.

In each production line $j \in [0,1]$, a single firm f may own multiple lines. A firm f is characterized by the countable set of lines for which it has leading technology $J_f^L \subseteq [0,1]$. Additionally, firm f can simultaneously be a follower and neck-to-neck, meaning firms produce at equal marginal costs, in other countable set of production lines respectively, $J_f^F, J_f^N \subseteq [0,1]$. The set J_f represents all the production lines in which firm f is active and is given by $J_f = J_f^L \cup J_f^F \cup J_f^N$ with the number of active product lines own by firm f is $n_f = \left|J_f^L\right| \in \mathbb{Z}_+$.

Intermediate Good Sector

The production function for firm f in line j at time t is expressed by

$$y_{fjt} = q_{fjt} x_{fjt}^{\alpha} l_{fjt}^{1-\alpha} \, \psi\Big((1-\xi)E_{fjt}, n_f\Big)^{1-\alpha},$$

where y_{fjt} is the output, q_{fjt} is productivity, x_{fjt} is amount of machines (tangible good) used. The law of motion satisfies $\dot{x}_{fjt} = I_{f,j,t}^X - \delta x_{fjt}$ and l_{fjt} is amount of labor used. The term $\alpha \in (0,1)$ is the output elasticity with respect to the machines and $\psi(.)$ represents labor productivity, satisfying $\psi'(.) > 0\,, \psi''(.) < 0$. The function $\psi(.)$ has two components: organizational capital and the number of active production lines that firm f has.

The term $(1-\xi)E_{fjt}$ shows that a $(1-\xi)$ fraction of relative embedded intangible assets is organizational capital, which increase labor productivity $\frac{\partial \psi(E_{fjt},\ .)}{\partial e_{fjt}} > 0$. Organizational capital represents the management skills of a firm, assuming that the distribution of these skills remains constant within each industry over time. However, the allocation of management skills between two firms changes based on their relative levels of organizational capital. When one firm owns more organizational capital, the constant distribution of management skills implies that its rival will have lower managerial skill levels. Therefore, both firms operate within a zero-sum game environment concerning management. Alternatively, if firm f has extensive organizational capital, its labor productivity will be higher due to superior management skills. Conversely, the labor efficiency of firm $-f$ will be lower owing to its comparatively inferior management skills.

The second component of the labor productivity function $\psi(.)$ is firm f's number of active production lines n_f. If firm f increases the number of its active production lines, this results in a decrease in the firm's labor productivity across each production line, under the assumption of the span of control (Lucas, 1978). The main rationale for the span of control assumption is that as a firm's size increases, its control over each production lines decreases. Therefore, $\frac{\partial \psi(.)}{\partial n_f} < 0$.

In summary, the multivariate labor productivity function $\psi(.)$ increases with organizational capital reflecting higher management skills and decreases with the number of production lines that the firm has due to management control difficulties. This situation underscores that adding a new production line may not always be advantageous due to reduced labor productivity across existing production lines. Moreover, while a firm may choose to expand its production lines, this typically results in diminishing profits and markup per production line. A detailed discussion of these dynamics will be provided in the equilibrium section.

In each intermediate sector j, two firms, denoted as $f \neq -f$, compete under a la Bertrand to sell their product. If $q_{fjt}\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha} > q_{-fjt}\psi((1-\xi)E_{-f}, n_{-f})^{1-\alpha}$, I refer to firm f as the leader in production line j and firm $-f$ as the follower. If $q_{fjt}\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha} = q_{-fjt}\psi((1-\xi)E_{-fjt}, n_{-f})^{1-\alpha}$, there is neck to neck to competition. To simplify notation, I define the marginal cost competitive advantage $\tau_{fjt} \equiv q_{fjt}\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha}$.

Transferable and Embedded Intangible Assets Evaluation

The Transferable assets evolve according to $q_{fjt} = \lambda^{m_{fjt}} q_{fj0}$, with the initial productivity level $q_{fj0} = 1$. Here, m_{fjt} represents the number of innovations in firm f and j denotes each machine line at time t. When a firm innovates between t and $t + \Delta t$, its transferable assets level increases by $\lambda > 1$,

$$q_{fj(t+\Delta t)} = \lambda q_{fjt}.$$

The transferable assets level differences between leader and follower can be expressed

$$\frac{q_{fjt}}{q_{-fjt}} = \frac{\lambda^{m_{fjt}}}{\lambda^{m_{-fjt}}} = \lambda^{(m_{fjt} - m_{-fjt})} = \lambda^{m}.$$

In a similar manner, embedded intangible assets differences can be expressed as

$$\frac{e_{fjt}}{e_{-fjt}} = \frac{\theta^{k_{fjt}}}{\theta^{k_{-fjt}}} = \theta^{(k_{fjt} - k_{-fjt})} = \theta^{k},$$

where e_{fjt} represents embedded intangible assets of firm f in industry j at time t and $\theta > 1$. The initial embedded value $e_{fj0} = 1$ and k_{fjt} denotes the number of innovations in embedded intangible assets of firm f.

Investment on R&D and Embedded Intangible Assets

The variables $I_{fjt}^{Emb}, I_{fjt}^{Int}, I_{fjt}^{Ex}$ represent firm f's investment in embedded intangible assets, internal transferable investment on its production line j, and external transferable investment on other production lines, respectively. If a firm is a leader in a particular production line, it can conduct an external innovation. Each unit of investment generates a successful innovation flow rates on internal transferable $\left(z_{fjt}^{Int}\right)$, external transferable $\left(z_{fjt}^{Ex}\right)$ and embedded assets $\left(z_{fjt}^{Emb}\right)$, respectively. Firm f internal and external investment on transferable intangible assets,

$$z_{fjt}^{Int} = \phi\left(I_{fjt}^{Int}\right) \Rightarrow G\left(z_{fjt}^{Int}\right) = I_{fjt}^{Int},$$

$$z_{fjt}^{Ex} = \tilde{\phi}\left(I_{fjt}^{Ex}, n_f\right) 1\left\{\sum_{j=1}^{n+1} \pi_j\left(n_f + 1\right) > \sum_{j=1}^{n} \pi_j\left(n_f\right)\right\}$$

$$\Rightarrow \tilde{G}\left(z_{fjt}^{Ex}, n_f\right) 1\left\{\sum_{j=1}^{n+1} \pi_j\left(n_f + 1\right) > \sum_{j=1}^{n} \pi_j\left(n_f\right)\right\} = I_{fjt}^{Ex},$$

with satisfies the condition,

$$I_{fjt}^{T} = I_{fjt}^{Int} + I_{fjt}^{Ex}.$$

Investment in embedded intangible assets is given by

$$z_{fjt}^{Emb} = \phi\left(I_{fjt}^{Emb}\right) \Rightarrow G\left(z_{fjt}^{Emb}\right) = I_{fjt}^{Emb}.$$

From the investment equations, $\phi(.)$ is continuously twice differentiable, satisfying $\phi'(.) > 0,\ \phi''(.) < 0$ and $\phi(0) < \infty$. The inverse function $G(.)$ is also twice differentiable with $G'(.) > 0, G''(.) > 0$. In external innovation, a firm invests in taking on a new production line if the additional line increases the firm's total profit. This decision is influenced by the fact that adding new production lines increases the marginal costs of all leading production lines due to the span of control problem. Additionally, the cost of innovating in new production lines escalates as firm size increases, represented by $\frac{\partial \tilde{G}\left(z_{fjt}^{Ex}, n_f\right)}{\partial n_f} > 0$. The representation of inverse function $\tilde{G}(.)$ stems from empirical observation 2 which indicates that larger firms exhibit lower transferable and embedded intensity compared to smaller firms. Smaller firms typically have higher transferable and embedded intensity because they can get greater benefits from adding new production lines. However, this benefit diminishes as their size increases.

Creative Destruction of Leader in Other Industry

If firm f is a leader in the production line j, it successfully makes an external innovation with flow rate z_{fjt}^{Ex}. The firm then randomly enters any production line j' and becomes a new producer if the condition

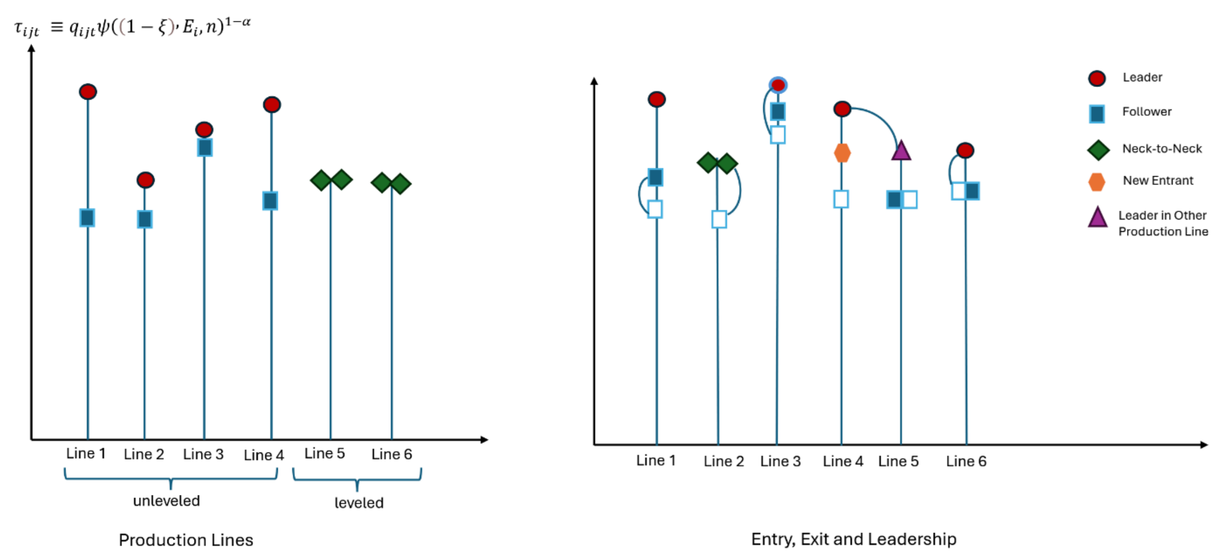
$$p_f^{Ex} = \mathbb{P}\left(E_j \geq E_{j'}\right)$$

is satisfied. After taking the production line, firm f's embedded intangible level in sector j' is E_j. If the relative embedded assets level of firm f in production lines j is high, the probability of taking on a new production line increase. The main economic interpretation is that even if firm f initiates creative destruction in a random production line j', this action alone is not sufficient to achieve leadership. Firm f's embedded intangible level must also be weakly higher than that of its rivals in entering the random production line j'.

Evaluation of Production Lines

Figure 3 illustrates potential scenarios in the market structure of production lines, encompassing firm entry, exit, and leadership dynamics. Each production line can exhibit either unleveled competition, where one firm is ahead of its rival, or levelled competition, indicating neck-and-neck competition. The key to maintaining leadership lies not only in productivity or the value of embedded assets alone, but in the total advantage they collectively confer. Therefore, the vertical axis $\tau_{fjt} = q_{fjt}\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha}$ represents the firm's total competitive level in production line j.

Figure 3: Production Lines Evaluation[4]



In an unleveled production line, the leader has the option to either make incremental innovations within its current production line or attempt to acquire new production lines in other sectors. For

[4] The illustration is a modified version of Akcigit and Ates (2021) Figure 12

example, in scenario four, the leader utilizes external investment to acquire production line five, leading to the exit of one of the follower firms from the market randomly. With a probability p^{Int}, a follower can make incremental innovations on its production line, whereas with a probability of $(1-p^{Int})$, it can make drastic innovations to quickly catch up to the leader's transferable asset level. Additionally, followers can quickly catch up with the leader's embedded asset value with a probability p^{Emb}, or incrementally increase their embedded assets with a probability $(1-p^{Emb})$.

In production line 3 follower is very close in terms of cost advantage τ_{-fjt}. For example, the follower might be one step ahead in terms of embedded assets but five steps behind in terms of transferable assets. In such cases, if firm $-f$ make drastic innovation on transferable intangibles, it can emerge as the new leader, as shown in line 3. Alternatively, the follower could achieve a neck-and-neck position with the leader (line 2) or, with incremental innovation, reduce the gap (line 1). Additionally, new entrants can also enter the market and become new followers (line 4). Lastly, if two firms are in neck-and-neck competition, one firm can increase either its productivity or its embedded asset level to escape competition and become the new leader (line 6).

Equilibrium

In this section, I describe the general equilibrium. I start with the static equilibrium of the model. Following this, I characterize the Markov Perfect equilibrium, including the relevant payoffs, value functions, and the evaluation of the distribution related to state variables. According to household optimal decision, the Euler equation expressed

$$\frac{\dot{C}_t}{C_t} = r_t - \rho.$$

The final good producer optimizes demand for each intermediate sector $j \in [0,1]$

$$y_{fjt} = \frac{\Upsilon_t}{p_{fjt}},$$

and p_{fjt} represent price of firm f's good in intermediate sector j. The first-order condition shows that, given the Cobb-Douglas nature of industry aggregation, firms cannot internalize the demand effect (brand value). However, an aggregate increase in brand value shifts the demand for the firm's products.

In the intermediate good sector, the firm converts one unit of final goods into a machine at a cost of R_t^x and the machine depreciated at a rate of δ each time period. Therefore, interest rate expressed as $r_t = R_t^x - \delta$. The marginal unit cost of intermediate goods producer is

$$MC_{fjt} = \left(\frac{r_t + \delta}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \frac{1}{\psi((1-\xi)E_{fjt}, n_f)^{1-\alpha}} \frac{1}{q_{fjt}}.$$

Under Bertrand equilibrium, leader sets price equal to marginal cost of the follower $p_{fjt} = MC_{-fjt}$ and equilibrium output expressed $y_{fjt} = \frac{\Upsilon_t}{MC_{-fjt}}$. If $\tau_{fjt} \geq \tau_{-fjt}$, intermediate good output $y_{fjt} > 0$; otherwise, it is 0.

The definition of profit is $(p_{fjt} - MC_{fjt})y_{fjt}$ and markup is $\frac{p_{fjt}}{MC_{fjt}}$ with equilibrium intermediate goods output and using transferable and embedded assets gaps $\frac{q_{fjt}}{q_{-fjt}} = \lambda^m$ and $E_f = \frac{e_{fjt}}{e_{fjt} + e_{-fjt}} = 1 + \theta^k$ which implies

$$\pi_{fjt} = \left[1 - \frac{\psi\left((1-\xi)(1+\theta^{-k}), n_{-f}\right)^{1-\alpha}}{\psi\left((1-\xi)(1+\theta^{k}), n_{f}\right)^{1-\alpha}} \frac{1}{\lambda^m}\right] \Upsilon_t,$$

$$\frac{p_{fjt}}{MC_{fjt}} = \frac{\psi\left((1-\xi)(1+\theta^{n}), n_{f}\right)^{1-\alpha}}{\psi\left((1-\xi)(1+\theta^{-n}), n_{-f}\right)^{1-\alpha}} \lambda^m.$$

The above equations show that Profit and markup can be reduced transferable assets λ^m and embedded assets θ^k and number of production line gap n. To simplify notation, markup can be represented with the cost advantage gap $\tau_{m,k,n} = \frac{\tau_{fjt}}{\tau_{-fjt}} = \frac{p_{fjt}}{MC_{fjt}}$. Firm f total profit and markup in active production lines follows as

$$\pi_{f,t} = \sum_{j \in J_f^L} \pi_{fjt}, \quad \frac{p_{ft}}{MC_{ft}} = \sum_{j \in J_f^L} \frac{p_{fjt}}{MC_{fjt}}.$$

Proposition 1: When a firm increases its production lines, its total markup and profit increase, however, the markup and profit of each production line decrease due to the span of control problem.

Proposition 2: Embedded intangible assets affect the steady state and influence the growth of output during the transitional period. The engine of growth comes from transferable intangible assets. Proof: See Appendix A.4

The first-order condition tangible assets in the intermediate goods problem implies

$$\frac{p_{fjt}\ \Upsilon_t q_{-fjt}\ \psi\left((1-\xi)E_{-fjt}, n_{-f}\right)^{1-\alpha}}{\left(\frac{r_t+\delta}{\alpha}\right)^{1+\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}} = x_{fjt}$$

which indicates that an increase in the embedded intangible assets gap suppresses the demand for tangible assets. Widening the relative embedded intangible assets gap increases the follower's marginal cost by decreasing labor productivity and decreases the leader's marginal cost due to increased labor productivity. Since the leader sets the price equal to the marginal cost of the follower, the price of the leader's product increases. Therefore, increasing the embedded intangible assets gap creates a sharp price and markup effect in that sector. In static equilibrium, a strong markup effect depresses output demand. One important remark here is that, although a follower does not supply goods in production line j, it can be a leader and supply goods in other production lines. The term n_{-f} represents the total number of active production lines for firm $-f$. The marginal cost of firm $-f$ in production line j increases as the number of its active production lines increases.

The Joint distribution of transferable and embedded intangible gaps along with production line gap n is defined as

$$\sum_{m=0}^{M}\sum_{k=0}^{K}\sum_{n=0}^{N}\mu_{m,k,n}(t) = 1.$$

The term $\{\mu_{m,k,n}(t)\}_{m,k,n=0}^{M,K,N}$ denote joint distribution of industries over different transferable and embedded assets and production line gaps. Transferable and embedded capital gap bounded with far future limit M, K and production line gap bounded naturally with external investment function equation. The focus on Markov Perfect Equilibria (MPE) allows to drop dependence on industry j. For this reason, I refer to $z_{m,k,n}^{Int}$ for the technological leader with m step ahead and by $z_{-m,k,n}^{Int}$ for a follower that is m step behind. A similar notation applies for embedded capital assets and external transferable investment $z^{Emb}_{m,k,n}$ and $z^{Ex}_{m,k,n}$, respectively. The list of decisions by leader and follower with technology gap m, embedded asset gap k and production line gap n at time t is denoted by $\Gamma_{m,k,n,t} = \{z^{I}_{m,k,n}, z^{Emb}_{m,k,n}, z^{Ex}_{m,k,n}, x_{ijt}, p_{ijt}, y_{ijt}\}$. An allocation in this economy is then given by time paths of decisions for a leader that is m, k and n steps ahead $\{\Gamma_{m,k,n,t}\}_{t=0}^{\infty}$, the time paths of transferable and embedded asset decision for a follower that m, k and n steps behind $\{\Gamma_{-m,-k,-n,t}\}_{t=0}^{\infty}$, time path of wages and interest rates$\{w_t, r_t\}_{t=0}^{\infty}$, and distribution of time paths of industry distribution over transferable, embedded assets gaps and production line n, $\{\mu_t\}_{t=0}^{\infty}$. The game consists of $\Gamma_{m,k,n,t}$ and $\Gamma_{-m,-k,-n,t}$ and MPE represent time paths $[\mathbf{\Gamma}_t^*, w_t^*, r_t^*, Y_t^*, X_t^*]$.

Value Functions

<u>Leader value function:</u>

$$
\begin{aligned}
r_t V_{m,k,n} - \dot{V}_{m,k,n} = \max_{z^{Int}_{m,k,n}, z^{Ex}_{m,k,n}, z^{Emb}_{m,k,n}} \Bigg\{ & \pi - G\left(z^{Int}_{m,k,n}\right) - G\left(z^{Emb}_{m,k,n}\right) - G\left(z^{Ex}_{m,k,n}, n\right) 1 \left\{ \sum_{j=1|j\in J_f^L}^{n+1} \pi_j(n+1) > \sum_{j=1|j\in J_f^L}^{n} \pi_j(n) \right\} \\
& + z^{Int}_{m,k,n}\left[V_{m+1,k,n} - V_{m,k,n}\right] + z^{Emb}_{m,k,n}\left[V_{m,k+1,n} - V_{m,k,n}\right] \\
& + (1-p^I) z^{Int}_{-m,-k,-n}\left[V_{m-1,k,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{m-1,k,n} > \tau_{-m+1,-k,-n}\right\} \\
& + p^I z^{Int}_{-m,-k,-n}\left[V_{0,k,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{0,k,n} > \tau_{-0,-k,-n}\right\} \\
& + (1-p^{Emb}) z^{Emb}_{-m,-k,-n}\left[V_{m,k-1,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{m,k-1,n} > \tau_{-m,-k+1,-n}\right\} \\
& + p^{Emb} z^{Emb}_{-m,-k,-n}\left[V_{m,0,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{m,0,n} > \tau_{-m,-0,-n}\right\} \\
& + (1-p^I) z^{Int}_{-m,-k,-n}\left[V^{-}_{m-1,k,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{m-1,k,n} \le \tau_{-m+1,-k,-n}\right\} \\
& + p^I z^{Int}_{-m,-k,-n}\left[V^{-}_{0,k,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{0,k,n} \le \tau_{-0,-k,-n}\right\} \\
& + (1-p^{Emb}) z^{Emb}_{-m,-k,-n}\left[V^{-}_{m,k-1,n} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{m,k-1,n} \le \tau_{-m,-k+1,-n}\right\} \\
& + z^{Emb}_{-m,-k,-n} p^{Emb}\left[V_{m,0,n-1} - V_{m,k,n}\right] \mathbb{I}\left\{\tau_{m,0,n} \le \tau_{-m,-0,-n}\right\} + \widetilde{z^{Int}_{-m,-n,0}} \widetilde{z^{Emb}_{-m,-n,0}}\left[V_{m-1,k-1,n} - V_{m,k,n}\right] \\
& - p^{Ex}_{-f} z^{Ex}_{-m,-k,-n} \left[V^{-}_{m-1,k,n} - V_{m,k,n}\right] + p^{Ex}_{f} z^{Ex}_{m,k,n}\left[V_{m,k,n+1} + V_{\bar{m},k,n+1} - V_{m,k,n}\right] \Bigg\}
\end{aligned}
$$

The left-hand side of the value function shows the return on the value function and its gain. On the right-hand side, the first term represents profit, while the subsequent terms describe the costs associated with investments in internal productivity, embedded intangible and external productivity investment respectively. The second term in the second line illustrates that, with a flow rate of $z^{Int}_{m,k,n}$, the productivity of the leader increases from m to $m+1$._The third line in second term with probability $(1-p^I)$ the follower can make incremental innovation and close productivity gap with one rung. In the fourth lines with probability p^I the follower can quick catch up the leader's productivity level. The same intuition applies to embedded asset improvement, with p^{Emb} the follower can make an incremental improvement and with $(1-p^{Emb})$ the follower can quick catch the leader, as shown in in lines five and six. However, even if the follower quickly catches up to the leader's productivity or embedded asset levels, it can still remain a follower. Alternatively, if the follower has higher (or equal) levels of one type of asset and quickly catches up to the leader's cost advantage in either embedded or transferable assets, it can become the new leader (or achieve a neck-to-neck position). Therefore, the follower's situation can change after making an innovation if it achieves a cost advantage $\tau_{m,k-1} \leq \tau_{-m,-k+1}$ or does not $\tau_{m,k-1,n} > \tau_{-m,-k+1,-n}$. For instance, in line four follower increase productivity level with drastic innovation, however, remains a follower because it couldn't gain the cost advantage. In line seven, eight, nine and first term in line ten show that the follower can either take leadership or become a neck to neck with its innovation. The terms V^{-}'s in the value functions indicate that after follower innovations leader loses the leaderships position. In line ten, second term show that with a flow rate of $\widetilde{z^{Int}_{-m,-n,0}}\widetilde{z^{Emb}_{-m,-n,0}}$ a new entrant improves productivity and embedded intangibles, narrowing the gaps by one level. The Last line shows that with $p^E_f\ z^{Ex}_{m,k,n}$, the leader can acquire a new production line with an expected return of $V_{\bar{m},k,n+1}$. Since the leader randomly enters any production line, its expectation on the productivity gap is the average productivity gap across all industries, denoted as $\bar{m}$. Conversely, $-p^E_{-f} z^{Ex}_{-m,-k,-n}$ allows leaders in other industries to acquire this production line.

<u>Follower value function:</u>

The main difference in the follower's value function compared to the leader is profit. Noticeably, in this scenario, the follower does not engage in production and consequently does not accrue any profits because the leader supplies all products. However, despite this lack of immediate returns, the firm adopts a forward-looking approach and allocates resources towards productivity and organizational capital. This strategic investment aims to first bridge the distance between itself and the leader and subsequently to surpass the leader through a series of successive innovations. The terms V^{+}'s in the value functions indicate that after follower innovations either taking a leaderships position or become a neck-to-neck position. Additionally, the last line shows that if new entrants or leaders in other industries innovate, the follower exits the market and its value function becomes zero.

$$
\begin{aligned}
r_t V_{-m,-k,-n} - \dot{V}_{-m,-k,-n} \\
= \max_{z^{Int}_{-m,-k,-n}, z^{Emb}_{-m,-k,-n}} \Big\{ & -G\left(z^{Int}_{-m,-k,-n}\right) - G\left(z^{Emb}_{-m,-k,-n}\right) + z^{Int}_{m,k,n}\left[V_{-m-1,-k,-n} - V_{-m,-k,-n}\right] \\
& + z^{Emb}_{m,k,n}\left[V_{-m,-k-1,-n} - V_{-m,-k,-n}\right] \\
& + (1-p^I) z^{Int}_{-m,-k,-n}\left[V_{-m+1,-k,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{m-1,k,n} > \tau_{-m+1,-k,-n}\} \\
& + p^I z^{Int}_{-m,-k,-n}\left[V_{-0,-k,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{0,k,n} > \tau_{-0,-k,-n}\} \\
& + (1-p^{Emb}) z^{Emb}_{-m,-k,-n}\left[V_{-m,-k+1,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{m,k-1,n} > \tau_{-m,-k+1,-n}\} \\
& + p^{Emb} z^{Emb}_{-m,-k,-n}\left[V_{-m,-0,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{m,0,n} > \tau_{-m,-0,-n}\} \\
& + (1-p^I) z^{Int}_{-m,-k,-n}\left[V^+_{-m+1,-k,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{m-1,k,n} \leq \tau_{-m+1,-k,-n}\} \\
& + p^I z^{Int}_{-m,-k,-n}\left[V^+_{0,-k,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{0,k,n} \leq \tau_{-0,-k,-n}\} \\
& + (1-p^{Emb}) z^{Emb}_{-m,-k,-n}\left[V^+_{-m,-k+1,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{m,k-1,n} \leq \tau_{-m,-k+1,-n}\} \\
& + p^{Emb} z^{Emb}_{-m,-k,-n}\left[V^+_{-m,0,-n} - V_{-m,-k,-n}\right]\mathbb{I}\{\tau_{m,0,n} \leq \tau_{-m,-0,-n}\} + \widetilde{z^{Int}_{-m,-n,0}}\widetilde{z^{Emb}_{-m,-n,0}}\left[0 - V_{-m,-k,-n}\right] \\
& - p^{Ex}_{-f} z^{E}_{-m,-k,-n}\left[0 - V_{-m,-k,-n}\right]\Big\}
\end{aligned}
$$

Neck-to-Neck value function:

In neck-to-neck competition both firms cannot generate profit and the only way to escape competition is to increase productivity or embedded intangible assets level. With flow rate of innovation $z^{Int}_{0,0,n}$ or $z^{Emb}_{0,0,n}$ a firm improves its productivity and embedded intangibles level, respectively. Alternatively, with flow rate $z^{Int}_{-0,-0,-n}$ and $z^{Emb}_{-0,-0,-n}$, its rival can make an innovation. The last line shows that a new entrant or leader in other industry can make an innovation and there is a 50% probability that one of the firms will exit the market.

$$
\begin{aligned}
r_t V_{0,0,n} - \dot{V}_{0,0,n} = \max_{z^{Int}_{0,0,n}, z^{Emb}_{0,0,n}} \Big\{ & -G\left(z^{Int}_{0,0,0}\right) - G\left(z^{Emb}_{0,00}\right) + z^{Int}_{0,0,n}\left[V_{1,0,n+1} - V_{0,0,n}\right] + z^{Emb}_{0,0,n}\left[V_{0,1,n} - V_{0,0,n}\right] \\
& + z^{Int}_{-0,-0,-n}\left[V_{-1,0,n} - V_{0,0,n}\right] + z^{Emb}_{-0,-0,-n}\left[V_{0,-1,n} - V_{0,0,n}\right] \\
& + \left(\widetilde{z^{Int}_{0,0,0}}\widetilde{z^{Emb}_{0,0,0}} + p^{Ex}_{-f} z^{E}_{-m,-k,-n}\right)\frac{1}{2}\left[0 - V_{0,0,n}\right]\Big\}
\end{aligned}
$$

New Entrant value function:

$$
\max_{\widetilde{z^{Int}_{-m,-k,0}}, \widetilde{z^{Emb}_{-m,-k,0}}} \left\{ -G\left(\widetilde{z^{Int}_{-m,-k,0}}\right) + -G\left(\widetilde{z^{Emb}_{-m,-k,0}}\right) + \widetilde{z^{Int}_{-m,-k,0}}\widetilde{z^{Emb}_{-m,-k,0}}\left[V_{-m+1,-k+1,0} - 0\right] \right\}
$$

$$
\max_{\widetilde{z^{Int}_{0,0,0}}, \widetilde{z^{Emb}_{0,0,0}}} \left\{ -G\left(\widetilde{z^{Int}_{0,0,0}}\right) - -G\left(\widetilde{z^{Emb}_{0,0,0}}\right) + \widetilde{z^{Int}_{0,0,0}}\widetilde{z^{Emb}_{0,0,0}}\left[V_{1,1,1} - 0\right] \right\}
$$

A new entrant enters the market successfully only if it innovates on both productivity and embedded intangible assets with flow rate of innovation $\widetilde{z^{I}_{-m,-k,0}}\widetilde{z^{Emb}_{-m,-k,0}}$ and become a new follower. If an industry is in a neck-to-neck position, a successful innovation make a new entrant the leader.

Evaluation of Distribution

$$\begin{aligned}
\frac{\mu_{m,k,n,(t+\Delta t)}-\mu_{m,k,n,t}}{\Delta t} \\
&= z^{Int}_{m-1,k,n,t}\,\mu_{m-1,k,n,t} + z^{Emb}_{m,k-1,n,t}\,\mu_{m,k-1,n,t} + z^{Ex}_{m,k,n-1,t}p^{Ex}_{f}\,\mu_{m,k,n-1,t} \\
&+ z^{Int}_{m-1,k-1,n-1,t}z^{Emb}_{m-1,k-1,n-1,t}z^{Ex}_{m-1,k-1,n-1,t}p^{Ex}_{f}\,\mu_{m-1,k-1,n-1,t} \\
&+ z^{Int}_{m-1,k-1,n,t}z^{Emb}_{m-1,k-1,n,t}\mu_{m-1,k-1,n,t} + z^{Emb}_{m,k-1,n-1,t}z^{Ex}_{m,k-1,n-1,t}p^{Ex}_{f}\,\mu_{m,k-1,n-1,t} \\
&+ z^{Int}_{m-1,k,n-1,t}z^{Ex}_{m-1,k,n-1,t}p^{Ex}_{f}\,\mu_{m-1,k,n-1,t} + z^{Ex}_{.,k,n-1,t}p^{Ex}_{f}\,\mu_{m-1,k,n-1,t} \\
&+ (1-p^{I})z^{Int}_{-m-1,k,n,t}\mu_{m+1,k,n,t} + (1-p^{Emb})z^{Emb}_{m,k-1,n,t}\mu_{m,k+1,n,t} \\
&+ \left((1-p^{I})(1-p^{Emb})z^{Int}_{-m-1,k-1,n,t}z^{Emb}_{m-1,k-1,n,t}\right)\mu_{m+1,k+1,n,t} \\
&- \left(z^{Int}_{m,k,n,t} + z^{Emb}_{m,k,n,t} + z^{Ex}_{m,k,n,t}p^{Ex}_{f} + z^{Int}_{-m,-k,-n,t} + z^{Emb}_{-m,-k,-n,t} + z^{Ex}_{.,-k,-n,t}p^{Ex}_{-f}\right)\mu_{m,k,n,t} + \frac{o(\Delta t)}{\Delta t}
\end{aligned}$$

Given a small time period Δt, the evaluation of the mass of industries with m, k and n gaps are based on the differences between inflows and outflows. The first line shows that leaders with $(m-1, k, n)$ gaps, $(m, k-1, n\ )$ gaps or $(m, k, n-1)$ gaps experience inflow through successful innovations in productivity, embedded assets or external innovation in productivity, respectively. The second line shows leader's joint success in innovation on internal and external productivity and embedded intangible assets with $(m-1, k-1, n-1)$ gaps. The third line and first term in fourth line illustrate inflow through joint successful innovations in internal productivity and embedded assets, embedded assets and external productivity or internal and external innovations. The second term in the fourth line shows that a firm with external innovation can make a new production line with (m, k, n) gaps; here, $z^{Ex}_{.,k,n-1,t}$ represents any productivity gap industries with k and $(n-1)$ gaps. The fifth line indicates that followers through incremental innovation close the gaps in productivity or embedded intangible assets thereby inflowing $\mu_{m,k,n,t}$ with gaps one step larger. The sixth line shows that follower joint successful innovation in internal productivity and embedded. Outflow can occur with a follower, a leader, or a leader in another industry within the mass of industries $\mu_{m,k,n,t}$ where one of them succeeds in innovating productivity, embedded assets or acquiring a new production line.

Steady State Investment

In the steady state return of value function must be same for both internal and external productivity and embedded intangible asset investments. Calculations of steady state value function and free entry condition see the details Appendix in A.6.

$$z_{m,k,n}^{Int} = z_{m,k,n}^{E}, \qquad z_{m,k,n}^{T} = z_{m,k,n}^{Int} + z_{m,k,n}^{Ex}$$

$$z_{m,k,n}^{T} = z_{m,k,n}^{Emb}$$

$$z_{-m,-k,-n}^{T} = z_{-m,-k,-n}^{Emb}$$

$$z_{0,0,n}^{T} = z_{0,0,n}^{Emb}$$

Proposition 3: In steady three type of assets grow same rate g. Proof: See Appendix A.7

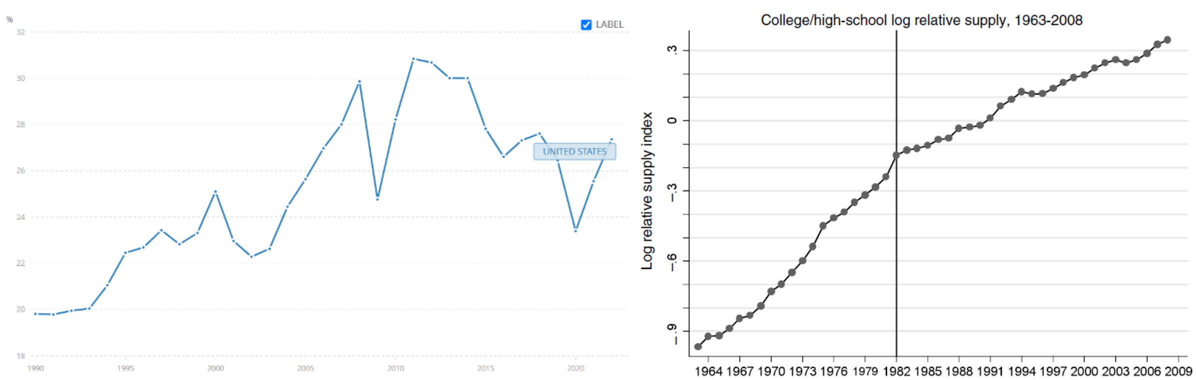
Dynamics

Firstly, under constant embedded intangible assets gap $\bar{k}$ and number of production line $\bar{n}$ with each one-step productivity gain, the difference in value functions decreases $v_{m+1,\bar{k},\bar{n}} - v_{m,\bar{k},\bar{n}} > v_{m+2,\bar{k},\bar{n}} - v_{m+1,\bar{k},\bar{n}}$ for all $m \geq 1$. As the leader increases its productivity level by one more step, its investment incentive decreases. An increase in the productivity gap results in a reduced investment in productivity. A similar rationale applies to the embedded intangible assets gap, assuming constant productivity and production lines, $\bar{m}, \bar{n},$ respectively. An increase in the embedded asset gap diminishes the incentive for investment embedded intangible assets in each step, $v_{\bar{m},k+1,\bar{n}} - v_{\bar{m},k,\bar{n}} > v_{\bar{m},k+2,\bar{n}} - v_{\bar{m},k+1,\bar{n}}$.

Extension of the Model

One explanation for the surge in intangible assets is attributed to globalization and skill-biased task specialization. Melitz and Redding (2023) discuss four channels through which trade influences innovation: market size, competition, spillover, and comparative advantage. Consequently, globalization has increased market size and competition, thereby amplifying the significance of intangible assets in achieving marginal cost advantages. The expansion in market size and heightened competition incentivizes firms to invest more in intangible assets. Figure 3 illustrates the trade share of the US relative to its GDP, a trend similar to the ratio of intangible to tangible assets depicted in Figure 1.

Figure 4: Trade %GDP and Supply of Skill Labor



Note: The source of the left figure is the World Bank, and the source of the right figure is Acemoglu and Autor (2011).

On the other hand, skill biased task specialization makes organizational capital of firm more important. Figure 4 illustrates the supply of skilled labor, which has shown a linear increase since the 1960s. The core idea behind skill-biased technological change is that the production of high-skill labor requires significant management expertise. As a result, firms find it necessary to increase their investments in organizational capital to oversee the production process effectively. This section explores which factors contribute to the increasing intangible assets. To achieve this, I implement directed technological change within the Schumpeterian step-by-step innovation model. For simplicity I assume that a leader can only be one step ahead from follower, external innovation does not allow, and each firm can be leader only in one production line. In this economy there are two types of labor high h and low skilled l and both of them supply inelastically. Final good output is expressed as

$$Y_t = \left(Y_{H,t}^{\sigma} + Y_{L,t}^{\sigma}\right)^{\frac{1}{\sigma}}.$$

From first order condition,

$$p_H = Y_{H,t}^{-\frac{1}{\sigma}} Y_t^{\frac{1}{\sigma}}, \quad p_L = Y_{L,t}^{-\frac{1}{\sigma}} Y_t^{\frac{1}{\sigma}},$$

and ratio of two high skill and low skill price expressed, $\frac{p_H}{p_L} = \frac{Y_{L,t}^{\frac{1}{\sigma}}}{Y_{H,t}^{\frac{1}{\sigma}}}$. Here $Y_{H,t}$ shows aggregate output produced with high skill labor and $Y_{l,t}$ shows aggregate output produced with low skilled labor. Price of high and low skill labor aggregate outputs are p_H and p_L respectively and $\sigma > 1$. The high and low type producer technology expressed,

$$Y_{H,t} = \exp\left(\int_0^1 \ln\left(A\left(\xi E_i^h\right) y_{ijt}^h\right)\right),$$

and

$$Y_{L,t} = \exp\left(\int_0^1 \ln\left(y_{ijt}^l\right)\right).$$

Here, y_{ijt}^h shows output produced with high skill labor in that sector and y_{ijt}^l shows output produced with low skill labor in another sector.

Assumption 1: Produced goods with low skill labor do not need embedded intangible assets, however, goods produced with high skill labor embedded intangible intensity in their production process.

In each intermediate good sector, there are two firms competing under a la Bertrand. From first order condition of high and low skill aggregator,

$$y_{ijt}^h = \frac{Y_{H,t} p_H}{p_h} \quad , \quad y_{ijt}^l = \frac{Y_{L,t} p_L}{p_l}.$$

Here p_l and p_h show price of low and high skill intermediate good, respectively. High Type Intermediate Good Problem express,

$$\min_{x_{ijt}, h_{ijt}} (r_t + \delta) x_{ijt} + w_t^h \, h_{ijt} \quad s.t$$

$$q_{ijt} x_{ijt}^{\alpha} h_{ijt}^{1-\alpha} \psi\left((1-\xi) E_i\right)^{1-\alpha} \le y^h.$$

Low Type Intermediate Good Problem equal to,

$$\min_{x_{ijt}, l_{ijt}} (r_t + \delta) x_{ijt} + w_t^l l_{ijt} \quad s.t$$

$$q_{ijt} x_{ijt}^{\alpha} l_{ijt}^{1-\alpha} \le y^l.$$

The marginal cost of high and low types

$$MC_i^h = \left(\frac{r_t + \delta}{\alpha}\right)^{\alpha} \left(\frac{w_t^h}{1-\alpha}\right)^{1-\alpha} \frac{1}{\psi\left((1-\xi)E_i^h\right)^{1-\alpha}} \frac{1}{q_{ijt}^h},$$

$$MC_i^l = \left(\frac{r_t + \delta}{\alpha}\right)^{\alpha} \left(\frac{w_t^l}{1-\alpha}\right)^{1-\alpha} \frac{1}{q_{ijt}^l}.$$

Profit expressed for both types

$$\pi_{ijt}^h = \left[1 - \frac{\psi\left((1-\xi)(1+\theta^{-n})\right)^{1-\alpha}}{\psi\left((1-\xi)(1+\theta^{n})\right)^{(1-\alpha)}} \frac{1}{\lambda^m}\right] Y_{H,t} p_H,$$

$$\pi_{ijt}^l = \left[1 - \frac{1}{\lambda^m}\right] Y_{L,t} p_l.$$

Markup for high and low type,

$$\mu^h = = \frac{\psi\left((1-\xi)(1+\theta^{n})\right)^{(1-\alpha)}}{\psi\left((1-\xi)(1+\theta^{-n})\right)^{1-\alpha}} \lambda^m,$$

$$\mu^l = \lambda^m.$$

Relative prices given by,

$$\left(\frac{p_H}{p_L}\right)^{\sigma} = \frac{p_L y_{ijt}^h p_h}{p_H y_{ijt}^l p_l},$$

$$\left(\frac{p_H}{p_L}\right)^{\sigma+1} = \frac{y_{ijt}^h}{y_{ijt}^l} \left(\frac{w_t^l}{w_t^h}\right)^{1-\alpha} \lambda^m \frac{1}{\psi\left((1-\xi)E_i^h\right)^{1-\alpha}}.$$

Under the assumption that both high and low types have the same productivity level and that their gap with their rivals is the same, the relative price function takes on the following form,

$$\left(\frac{p_H}{p_L}\right)^{\sigma+1} = \frac{y_{ijt}^h}{y_{ijt}^l} \left(\frac{w_t^l}{w_t^h}\right)^{1-\alpha} \frac{1}{\psi\left((1-\xi)E_i^h\right)^{1-\alpha}}.$$

Using demand of y_{ijt}^h and y_{ijt}^l expressed,

$$\left(\frac{p_H}{p_L}\right)^{\sigma+1} = \left(\frac{x_{ijt}^h}{x_{ijt}^l}\right)^{\alpha} \left(\frac{h}{l}\right)^{1-\alpha} \left(\frac{w_t^l}{w_t^h}\right)^{1-\alpha} \frac{1}{\psi\left((1-\xi)E_i^h\right)^{1-\alpha}}.$$

Since wage ratio

$$\left(\frac{w_{ss}^l}{w_{ss}^h}\right)^{1-\alpha} = \frac{Y_{L,t}p_L}{Y_{H,t}p_H} \frac{1}{\psi\left((1-\xi)E_i^h\right)^{1-\alpha}},$$

relative prices derive,

$$\frac{p_H}{p_L} = \left(\frac{x_{ijt}^h}{x_{ijt}^l}\right)^{2\alpha} \left(\frac{h}{l}\right)^{2(1-\alpha)} \frac{1}{\psi\left((1-\xi)E_i^h\right)^{(1-\alpha)}}.$$

For High and Low Type HJB in the Steady State

Here $f = \{High, Low\}$ and I introduce competition based on Aghion et al. (2005). Under a la Bertrand, price of leader in that industry must be marginal cost of the follower. Without collusion profits be $\pi_1^f > 0$ and $\pi_{-1}^f = 0$. Now suppose that two firms colluded to increase profit and they can together act like the leader in an unleveled sector and each earning profit equal to $\frac{\pi_1^f}{2}$. However, the leader does not incentive to share its profit. On the other hand, under there is a neck-to-neck competition both firms have incentive to collude, and I assume that $\pi_0^f = (1-\Delta)\pi_1^f$, $\frac{1}{2} \leq \Delta \leq 1$. Here Δ shows product market competition and $(1-\Delta)$ fraction of leader's profit that the leveled firm can attain through collusion.

Leader Value Function:

$$\rho v_{1,1}^f - \dot{v}_{1,1}^f = \max_{z_{1,1}^I, z_{1,1}^E} \{\pi_1^f - G(z_{1,1}^I) - G(z_{1,1}^{Emb}) + (z_{1,1}^I + z_{1,1}^E)[v_{1,1}^f - v_{1,1}^f] + (z_{-1,-1}^I + z_{-1,-1}^{Emb})[v_{0,0}^f - v_{-1,-1}^f]\}$$

Follower Value Function:

$$\rho v_{-1,-1}^f - \dot{v}_{-1,-1}^f = \max_{z_{-1,-1}^I, z_{-1,-1}^E} \{\pi_{-1}^f - G(z_{-1,-1}^I) - G(z_{-1,-1}^{Emb}) + (z_{-1,-1}^I + z_{-1,-1}^{Emb})[v_{0,0}^f - v_{-1,-1}^f] + (z_{1,1}^I + z_{1,1}^{Emb})[v_{1,1}^f - v_{1,1}^f]\}$$

Neck-to-Neck Competition Value Function:

$$\rho v_{0,0}^{f} - \dot{v}_{0,0}^{f} = \max_{z_{-1,-1}^{I}, z_{-1,-1}^{E}} \{\pi_{0}^{f} - G(z_{0,0}^{I}) - G(z_{0,0}^{Emb}) + (z_{0,0}^{I} + z_{0,0}^{E})[v_{1,1}^{f} - v_{0,0}^{f}] + (z_{-0,-0}^{I} + z_{-0,-0}^{Emb})[v_{-1,-1}^{f} - v_{-0,-0}^{f}]\}$$

In this model shifted $\left(\frac{h}{l}\right)^{2(1-\alpha)}$ show increase skill biased technological change. On the globalization effect, shifted Y_t catch the increase market size and increase Δ show competition effects on intangible assets.

Conclusion

In this paper, I implement a step-by-step innovation model to provide a unified explanation of stylized facts regarding intangible assets and their heterogeneity. The increasing ratio of intangible to tangible assets is driven by both transferable and embedded intangible assets. Transferable intangible assets contribute to output growth, while embedded intangible assets affect long-term outcomes. Although a firm's profit and markup increase with the acquisition of new production lines, the profit and markup of each individual production line decrease due to the assumption of span of control. In extending the model, I discuss two possible reasons for the increase in intangible assets: skill-biased technological change and globalization. To understand the impact of these factors on the rising intangible-to-tangible ratio, I apply the step-by-step innovation model within a framework of directed technological change.

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